

SHAPE & SPACE

MATHS MATTERS INSET 2015

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Teachers**

SHAPES, PATTERNS AND SYMMETRY AROUND US



Shape and Space

The mathematics curriculum provides opportunities for children to:

- gain knowledge of **geometrical relations in two or three dimensions** and recognise and appreciate shapes in the **environment**.
- develop **spatial awareness** and the ability to **recognise and make use** of the geometrical properties and objects.
- develop the ability to use **geometrical models as aids** to solving practical problems in time and space.

Mathematics: a Revised Syllabus for Primary Schools (June 2014) p.7

MATHEMATICS

a revised Syllabus for Primary Schools



primary
Mathematics
support team

June 2014

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Department of Curriculum Management - Malta

Shape and Space in real life situations...

Solving problems is a practical art, like swimming or skiing or playing the piano: you can learn it only by **imitation** and **practice**...if you wish to learn swimming you have to go into the water, and if you wish to become a **problem solver** you have to solve problems.



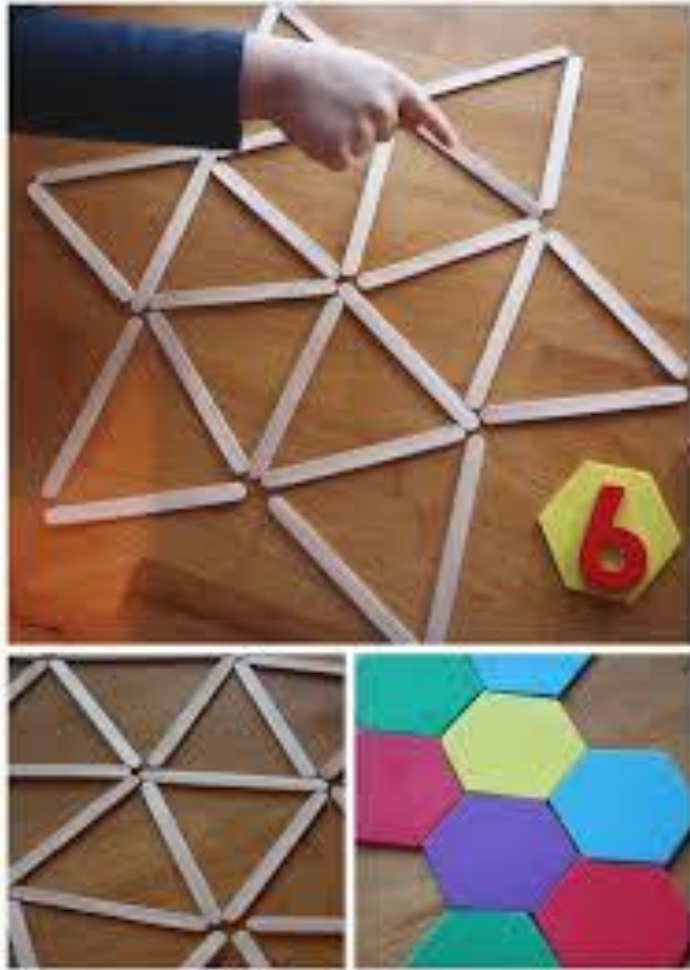
From *Mathematical Discovery* by George Polya

Adopting an investigative approach

One way to encourage pupils to think mathematically for themselves is to adopt an **investigative approach** to learning, where pupils are given **time**, **space** and **resources** to find things out for themselves while the teacher plays a supportive encouraging role.



Adopting an investigative approach



While concerns regarding time constraints and coverage of the curriculum should be recognised, it is generally accepted that children's **understanding** and **retention** will be **strengthened** as a result of an investigative approach, and so opportunities should be incorporated into the child's learning experience.

An Inquiry or cross-curricular approach

This type of **open-ended enquiry** can often usefully address problems identified by the students themselves, or may be a theme-based investigation.

In this type of enquiry the boundaries between mathematics and other curricular areas will often be **blurred**, as indeed they usually are in **real-life situations**.



8 KEY COMPETENCES



Communication
in the mother tongue



Communication
in foreign languages



Mathematical competence
and basic competences in
science and technology



Digital competence



Learning to learn



Social and civic
competences



Sense of initiative
and entrepreneurship



Cultural awareness
and expression



7 TRANSVERSAL SKILLS



Critical thinking



Creativity



Initiative



Problem-solving



Risk assessment



Decision-taking



Constructive management
of feelings

Strategies for effective Problem-solving

- ✓ **Look for a pattern**
- ✓ **Make a model (or use apparatus/equipment)**
- ✓ **Draw a picture/diagram**
- ✓ **Work together**
- ✓ **Guess, check & improve**
- ✓ **Act it out**
- ✓ **Produce an organised list/table**
- ✓ **Reason logically / systematically**
- ✓ **Try a simpler case**
- ✓ **Work backwards**

BLOOM'S TAXONOMY (REVISED)

CREATING

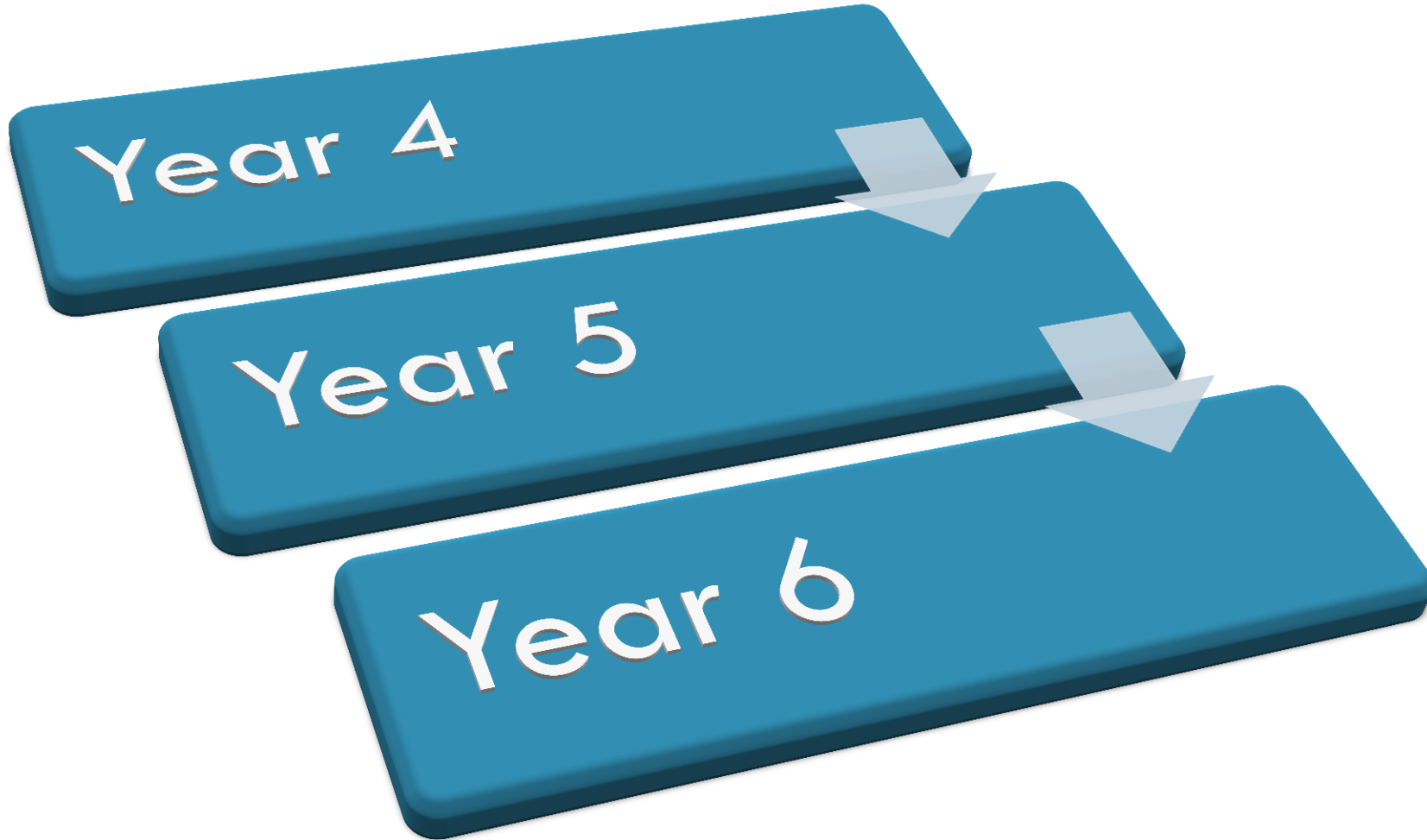
EVALUATING

ANALYZING

APPLYING

UNDERSTANDING

REMEMBERING



Learning Outcomes

J: SHAPES AND SYMMETRY

There is evidence of geometry everywhere. Buildings, planes, cars and maps all use geometry. For example, the home you live in is made of basic geometry shapes and some skyscrapers have windows made of rectangles and squares. Very often these structures are also symmetrical.

Symmetry can be seen almost everywhere in daily life. The human body is an example of symmetry: the kidney, the lungs the brain and to some extent even the face is.

Being able to understand the basic properties of 2-D and 3-D shapes, to draw shapes and to create your own patterns, whether symmetrical or not, will help you appreciate better the world we live in and will probably help you in your future career should you wish to become an engineer, a doctor, a scientist, a designer or a mechanic. However there are many other occupations that entail competence in geometry.

The above are only a few life situations where understanding of shapes and symmetry is important. Helping our children understand and appreciate this may be fruitful.

YEAR 4

LEARNING OUTCOMES Children will be able to:		KEY VOCABULARY	OPPORTUNITIES Children should be given a range of opportunities such as:
J.4.1	classify and describe 2-D (flat) and 3-D (solid) shapes, referring to properties, such as reflective symmetry, the number or shapes of faces, the number of sides/edges and vertices, whether sides/edges are the same length, whether or not angles are right angles.	two-dimensional (2-D) three-dimensional (3-D) semi-circle side vertex / vertices faces edges angles base	<ul style="list-style-type: none"> • handling 2-D (flat) and 3-D (solid) shapes and creating other shapes with them (e.g. exploring the different shapes that can be made from four cubes). • explaining the differences and similarities of 2-D (flat) and 3-D (solid) shapes, in response to questions such as "Which 3-D (solid) shapes have the same number of faces?", using vocabulary related to properties of shapes. • comparing and contrasting 2-D (flat) and 3-D (solid) shapes by stating their properties. • identifying right-angles in 2-D (flat) and 3-D (solid) shapes. • finding corresponding 2-D (flat) and 3-D (solid) shapes in the surrounding environment. • looking for symmetrical shapes or patterns in the environment (e.g. tiles, curtains, furniture and clothes). • identifying pictures or objects which have more than one line of symmetry. • continuing the reflective symmetry of a given diagram or object from real life. • participating in investigational maths activities that require problem-solving and allowing for further exploration of 2-D (flat) and 3-D (solid) shapes. e.g. Investigate the number of faces you can see when you arrange three cubes in different ways. • creating print patterns with faces of 3-D shapes and talk about them.
J.4.2	make and describe shapes and patterns.	right-angle symmetrical reflective symmetry	
J.4.3	identify and sketch lines of symmetry in simple shapes, and recognise shapes with no lines of symmetry.		
J.4.4	sketch the reflection of a simple shape in a mirror line along one edge.		

YEAR 5

LEARNING OUTCOMES Children will be able to:		KEY VOCABULARY	OPPORTUNITIES Children should be given a range of opportunities such as:
J.5.1	classify polygons using criteria.	Polygons regular irregular two-dimensional three-dimensional pentagon hexagon octagon straight side vertex / vertices edges faces base nets equilateral isosceles horizontal vertical diagonal fold line	<ul style="list-style-type: none"> • describing and visualising 2-D (flat) and 3-D (solid) shapes. • using geoboards and square paper to create and draw polygons. • recognising that polygons can be both regular and irregular, e.g. a pentagon can be any shape having 5 straight sides. • constructing 3-D (solid) shapes by using their corresponding nets or templates. • describing what a polygon is and identifying polygons from a given number of diagrams. • using paper folding and cutting to create equilateral and isosceles triangles. • working in groups and use technological equipment such as probots to create various polygons by giving a series of instructions. • comparing and contrasting the properties of equilateral and isosceles triangles. • creating diagrams and patterns using equilateral and isosceles triangles. • identifying horizontal and vertical lines in real life, e.g. on flags, furniture, doors and tiles. • using vertical and horizontal terms during art and craft activities, such as paper weaving craft.
J.5.2	visualise 3-D (solid) shapes from 2-D drawings and identify simple nets of solid shapes.		
J.5.3	recognise equilateral and isosceles triangles.		
J.5.4	recognise simple examples of horizontal and vertical lines.		

YEAR 6

LEARNING OUTCOMES Children will be able to:		KEY VOCABULARY	OPPORTUNITIES Children should be given a range of opportunities such as:
J.6.1	classify triangles using criteria.	reflective symmetry line of symmetry polygons regular irregular	<ul style="list-style-type: none"> stating the properties of different triangles, namely the equilateral, isosceles, and scalene triangles, by referring to the length of sides, their angles and lines of symmetry. identifying the lines of symmetry of a given polygon using shape templates and folding.
J.6.2	visualise 3-D shapes from 2-D drawings and identify different nets for a closed cube and an open cube.		
J.6.3	recognise reflective symmetry in regular polygons, and recognise where a shape will be after reflection in a mirror line parallel to one side.	two-dimensional three-dimensional pentagon, hexagon, octagon straight side side vertex / vertices edges faces base nets equilateral, isosceles, scalene right-angled triangle horizontal, vertical diagonal fold line	<ul style="list-style-type: none"> working in groups and use technological equipment such as probots to create various polygons by giving a series of instructions. exploring that regular polygons have the same number of lines of symmetry as the number of sides and the number of vertices. extend the understanding of regular polygons by observing polygons in real life and finding their lines of symmetry. using squared paper to complete the reflection of a polygon on the other side of a mirror line. exploring polygons and symmetrical patterns using polygons during interactive onscreen activities. describing polygons by referring to the sides, e.g. horizontal or vertical. participating in investigational maths activities that require problem-solving and allow for further exploration of 2-D (flat) and 3-D (solid) shapes.
J.6.4	complete symmetrical patterns with two lines of symmetry at right angles.		

K: Position, Direction and Angles

Being able to read a map to follow and/or give directions are functional skills. Distinguishing between left and right, between clockwise and anticlockwise turns and among the eight compass points will equip you with these skills. A pilot, a sailor, a fisherman, a policeman... and any driver need to have a good grasp of these skills. Furthermore engineers, architects, product designers use knowledge of angles daily.

The above are only some instances where position, direction and angles are important. Helping our children understand and appreciate these through their own experiences may be fruitful.

YEAR 4

LEARNING OUTCOMES Children will be able to:		KEY VOCABULARY	OPPORTUNITIES Children should be given a range of opportunities such as:
K.4.1	read and write the vocabulary related to position, direction and movement	north (N) south (S) east (E) west (W)	<ul style="list-style-type: none"> • describing and finding the position on a grid of squares with rows and columns labelled (e.g. Battleship, cinema seating, vending machines). • using a compass, recognising that a direction compass has a pointer which always points to the North. • using a map (e.g. describe points in relation to one another using the four-point compass). • comparing angles with a right angle in shapes and/or environment using a right-angle measure (template). • recognising that a straight line is equivalent to two right angles.
K.4.2	locate position on a grid with labelled rows and columns	map	
K.4.3	recognise and use the four-point compass directions.	turn	
K.4.4	make and describe right-angled turns, including turns between the four compass points (clockwise / anticlockwise).	clockwise anticlockwise right angle half turn quarter turn direction	
K.4.5	identify right angles in 2-D (flat) shapes and the environment.	direction compass angle row column grid position	

YEAR 5

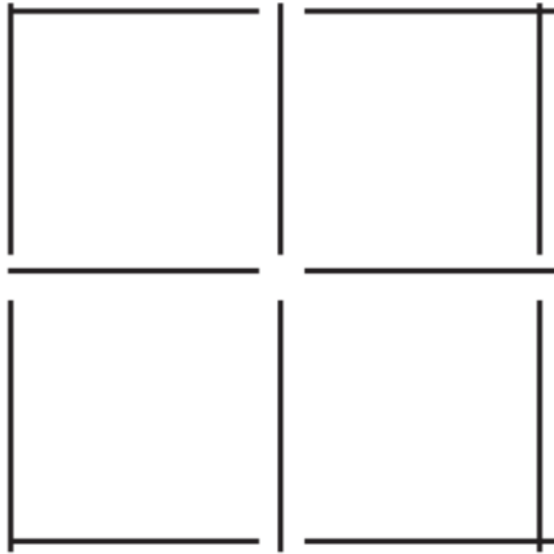
LEARNING OUTCOMES Children will be able to:		KEY VOCABULARY	OPPORTUNITIES Children should be given a range of opportunities such as:
K.5.1	recognise and use the eight compass directions.	direction/direction compass north/south/west/east north-east/north-west south-east/south-west halfway angle	<ul style="list-style-type: none"> • using a compass. • using a map (e.g. describe points in relation to one another using the four-point compass). • knowing that a degree is another measure of an angle and the shorthand for 'degree' is $^{\circ}$. • investigating the relationship between right angles and degrees
K.5.2	know that angles are measured in degrees.		
K.5.3	make and measure clockwise and anticlockwise turns (in degrees		
	and right angles).	clockwise turn half turn quarter turn degree ($45^{\circ}, 90^{\circ}, 180^{\circ}, 270^{\circ}, 360^{\circ}$) clockwise grid/grid line	<p>(e.g. 1 right angle = 90°, half a right angle = 45°, 3 right angles = 270°.....)</p> <ul style="list-style-type: none"> • investigating the size of the angle in degrees and right angles made by clock hands (e.g. 5 minute intervals). • using Pro-bot and/or Constructa-bot or other roamers to create angles. • planning and designing treasure/scavenger hunts using positional and/or directional vocabulary including angles.
K.5.4	order angles less than 180° .		

YEAR 6

LEARNING OUTCOMES Children will be able to:		KEY VOCABULARY or	OPPORTUNITIES Children should be given a range of opportunities such as:
K.6.1	understand the eight compass-point directions.	angle/right angle degrees ($^{\circ}$) direction/direction compass north/south/west/east north-east/north-west south-east/south-west halfway turn half turn quarter turn clockwise/anticlockwise grid/grid line protractor zero line centre scale acute/obtuse	<ul style="list-style-type: none"> • reinforcing the use of a compass. • constructing a wind vane, understanding the parts of a wind vane, understanding that wind vanes are used to measure wind direction and telling the wind direction from their own wind vane. • using the language of direction to guide a partner through a maze or to a place. • using a map (e.g. describe points in relation to one another using the four-point compass). • following and giving instructions involving distances by interpreting simple scales. • using Pro-bot and/or Constructa-bot or other roamers to programme commands/instructions to draw shapes/graphics to practice angles, directions and movement. • calculating one angle of a triangle, given the other two. • calculating a missing angle around a point, given the other/s. • using a protractor.
K.6.2	use angle measure in degrees		
K.6.3	identify, estimate and order acute and obtuse angles.		
K.6.4	estimate, measure and draw angles (acute and obtuse) in degrees to the nearest 5° using a protractor.		
K.6.5	understand that: <ul style="list-style-type: none"> • the sum of the angles of a triangle is 180°. • the angles on a straight line add up to 180°. • the angles around a point add up to 360°. 		

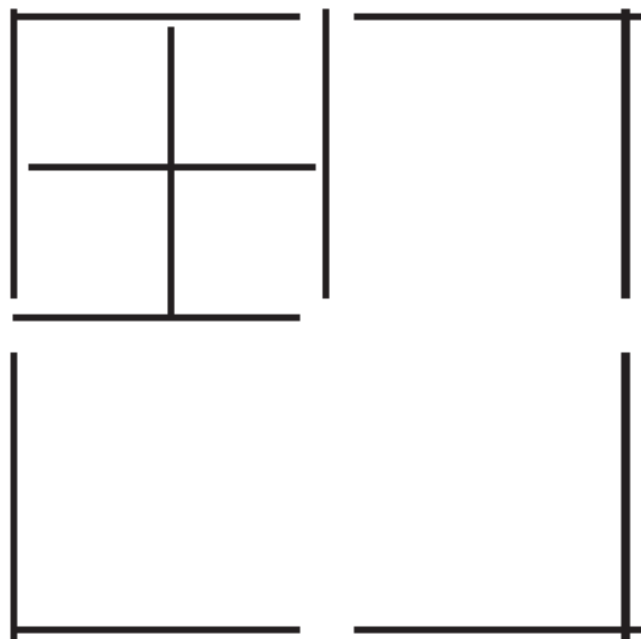
This is a 2 by 2 square.

Can you move 2 matches to make 6 squares?



starter activity

SOLUTION



solution

Why visualise?

- To step into a problem
- To model
- To plan ahead

Instances when we can visualise:

- Visualising 3D shapes (edges, vertices, faces)
- Visualising 2D shapes within 2D shapes.
- Estimating the total number of cubes stacked on top of each other by looking at a 2D image.
- Continuing a pattern or a reflection on shapes.
- Predicting a 3D from its net.

visualisation

The local council's emblem is shown in the picture below.

21. Each **vertex** on the star is joined together using a straight line. What shape does it become?

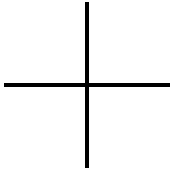


- Ask the pupils to visualise.
- What other shapes can they see inside the pentagon?
- Count the number of triangles.

The new shape is a _____.

Task 6

Use the Repeat button to create a plus sign. You can move forward and backward.

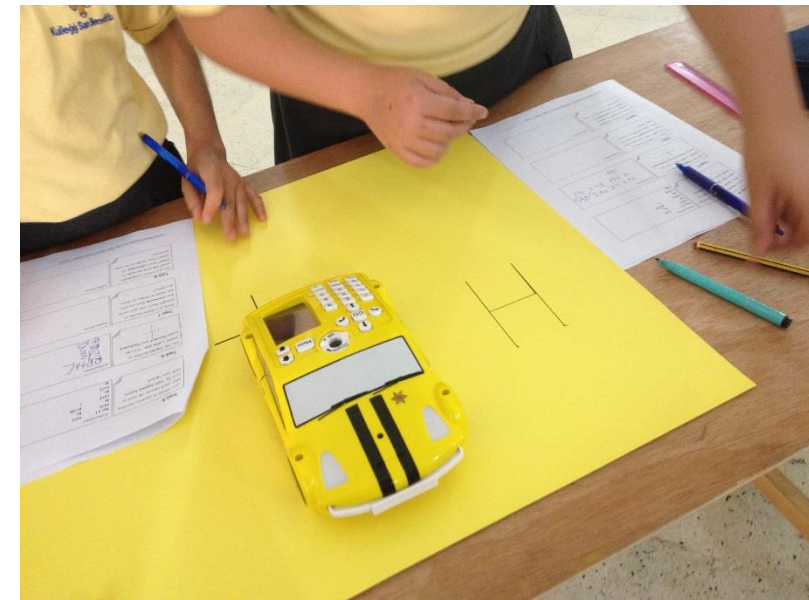


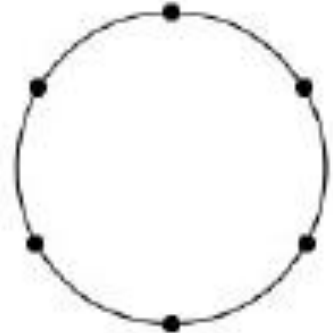
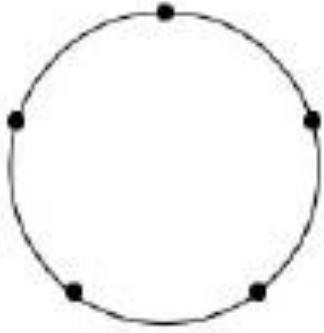
Instructions:

Task 7

Think of a letter (made of straight lines). Can you write the commands that the Pro-bot needs to follow to draw this letter?

Instructions:





- Different shapes can be made by joining some or all of the points in each diagram.
- How many isosceles triangles can be constructed on the four-point diagram?

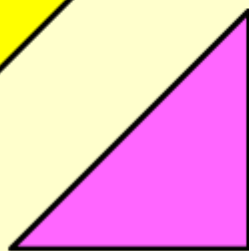
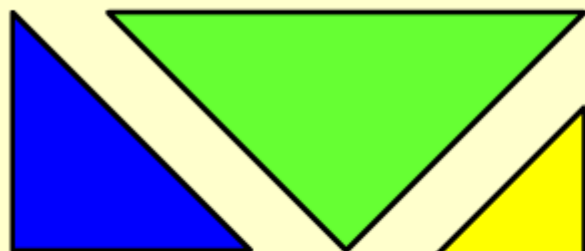
The image shows an interactive geometry software interface. On the left, a yellow circle has four green dots (pegs) placed on its circumference at the top, bottom, left, and right positions. To the right of the circle is a yellow table with three columns and one row. Each column contains a vertical line extending downwards from a horizontal top line. Above each vertical line is a colored symbol: a purple symbol on the left, a blue symbol in the middle, and a green symbol on the right. In the bottom-left corner, there is a control panel with the text "Pegs on Circle" above a numerical input field containing the number "4" and a small up/down arrow button.

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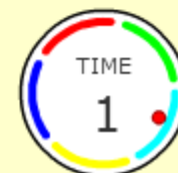
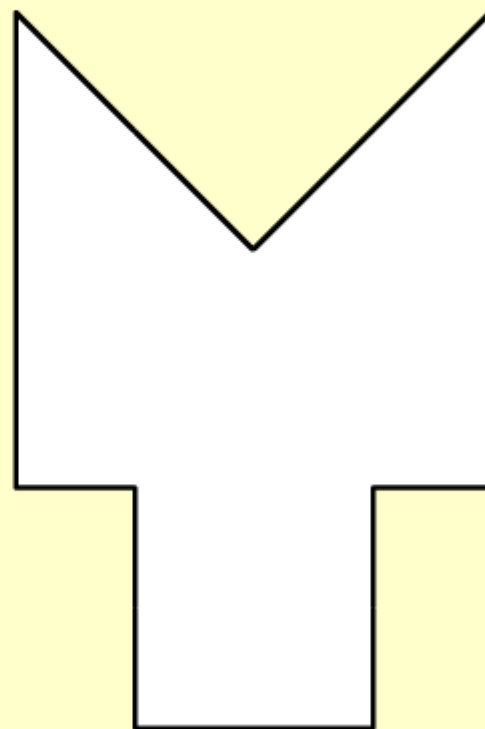
Pegs on Circle

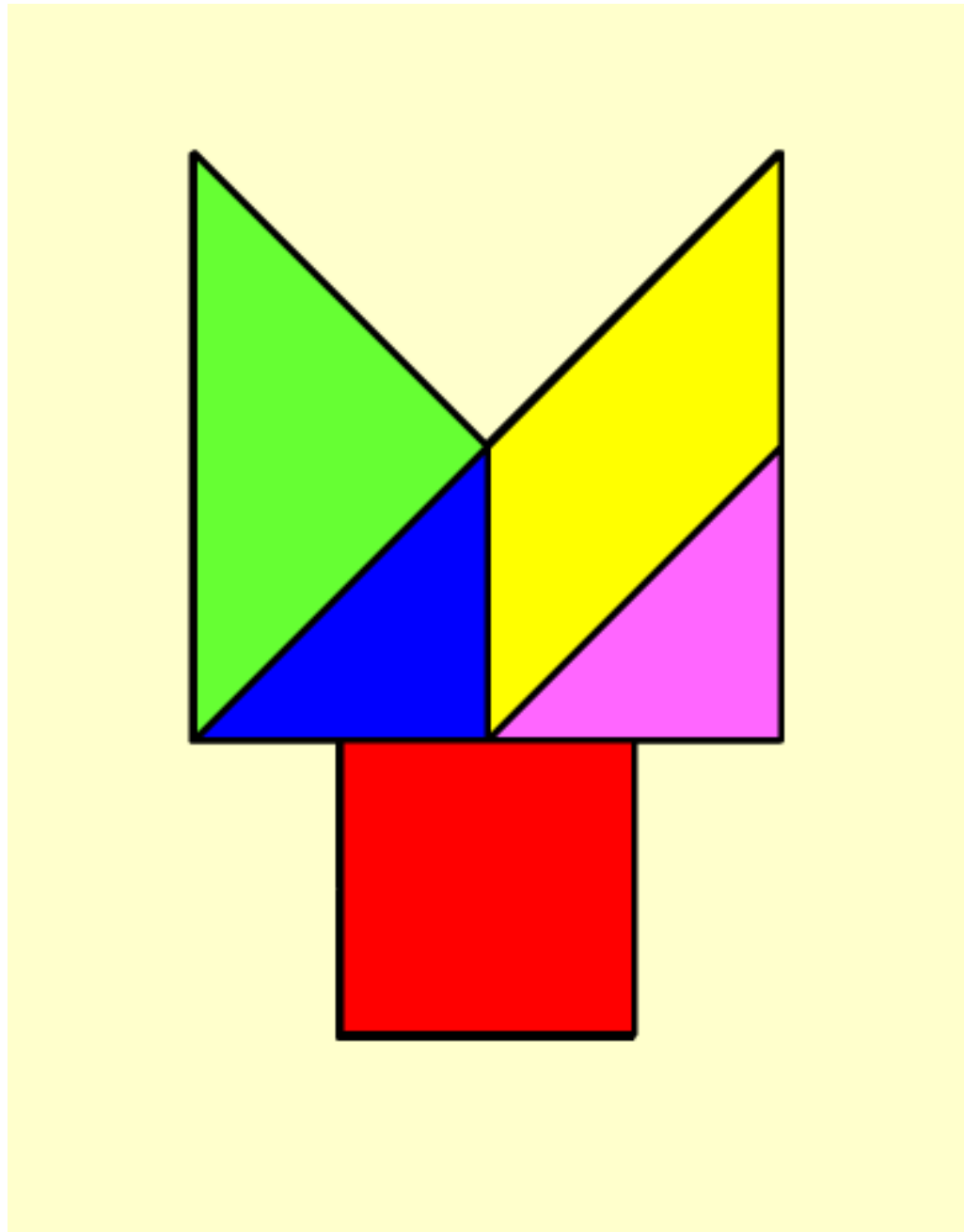
4

Tangram Game A

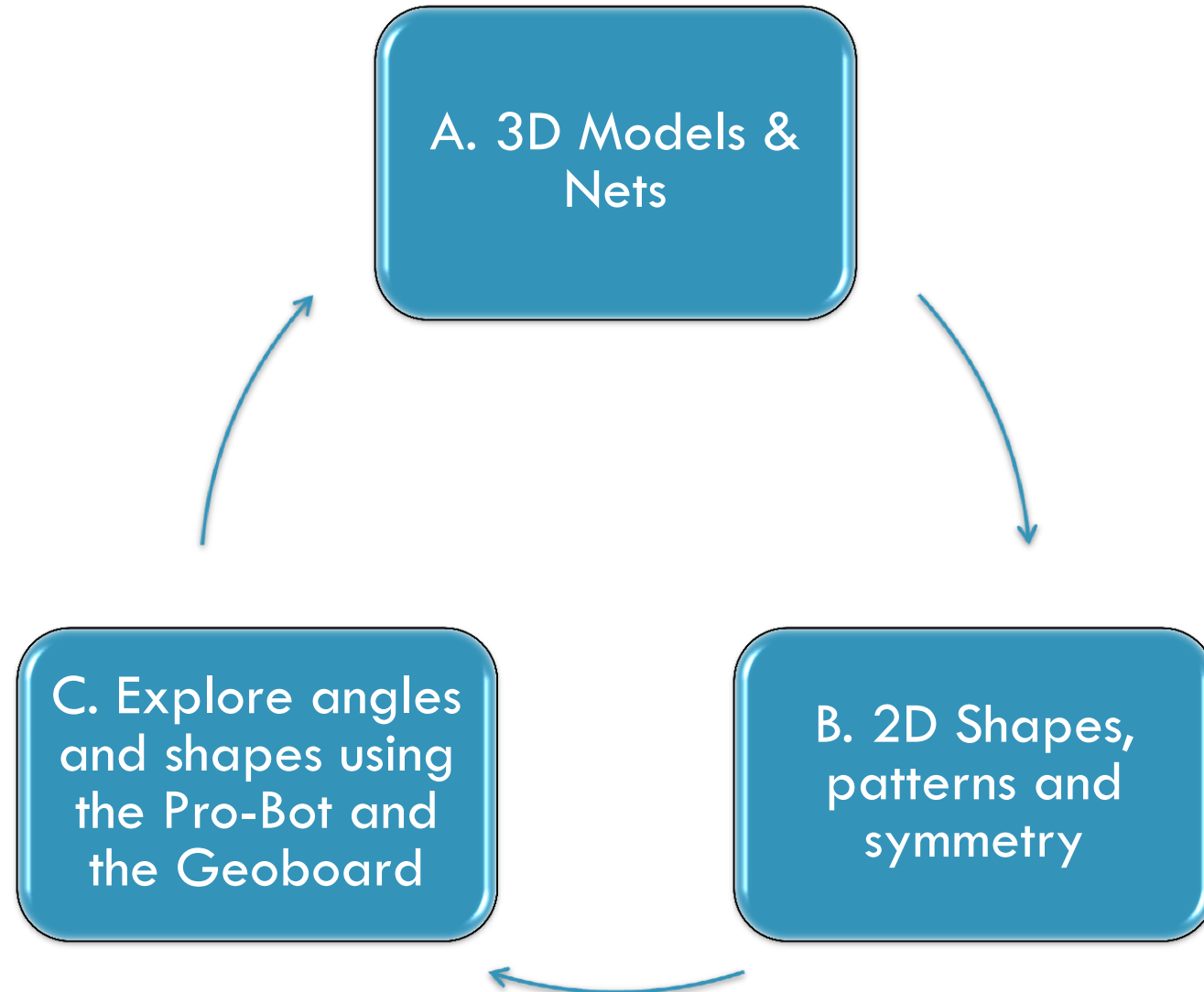


S1/4
-k-





Workshops (20 minutes x 3)



- Is there an idea which you will surely try in your classroom and why?
- Name one thing you observed that struck you, while carrying out one of the activities.

Some feedback please!



Patterned tiles



What is the shape of the tile?

What shapes can you see on the tile?

Draw the shapes you can see on an empty square template.

What 3D shape is created through the pattern?

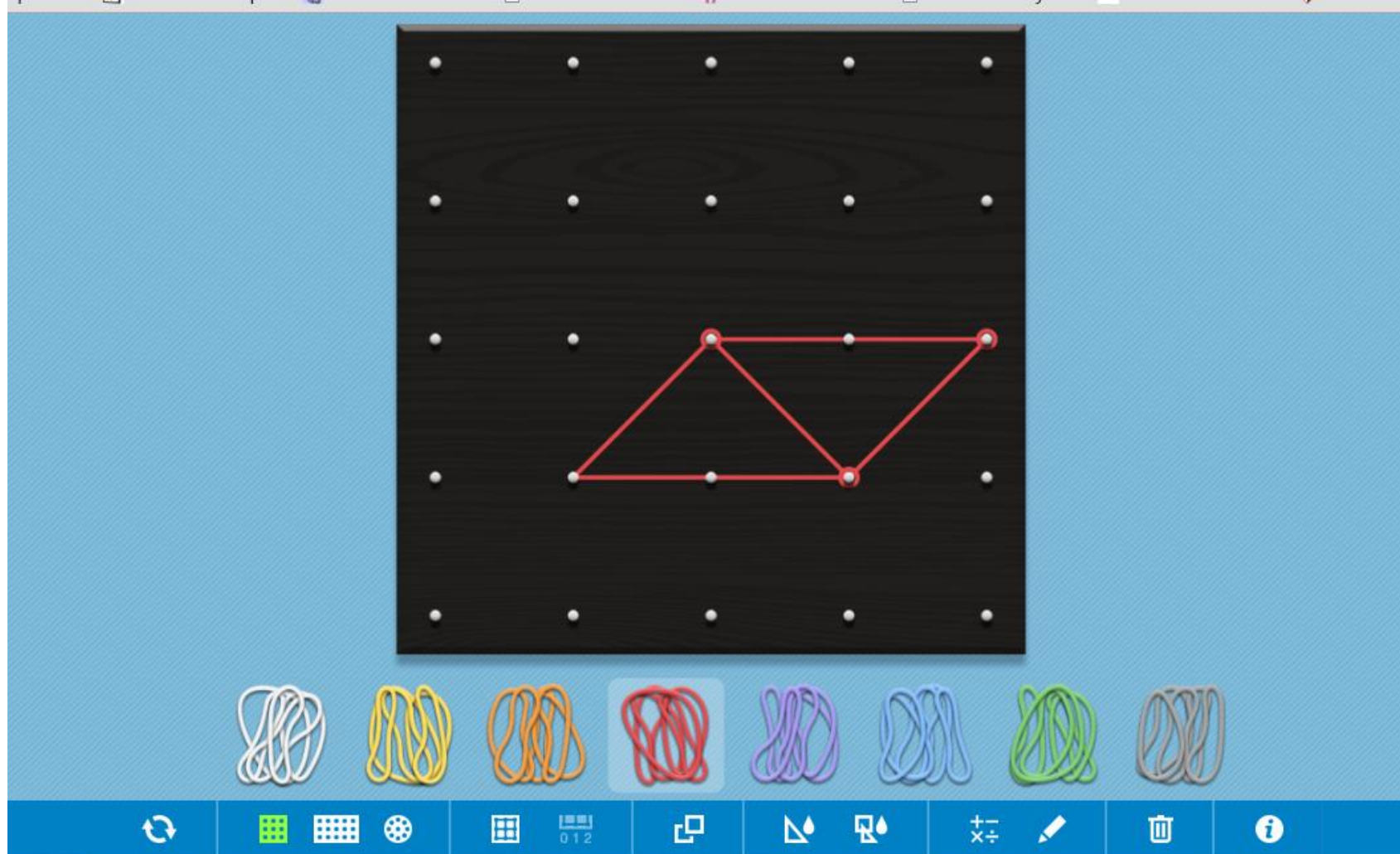
What can pupils gain through such an activity?

Can you create your own design to make a pattern of cubes?

Which other shapes can we use for tiling?

one square tile

<http://www.mathlearningcenter.org/web-apps/geoboard/>



NOTHING